Distance Learning Initiative

Introduction to Robotics

Robotic Arm Link Velocities - Example

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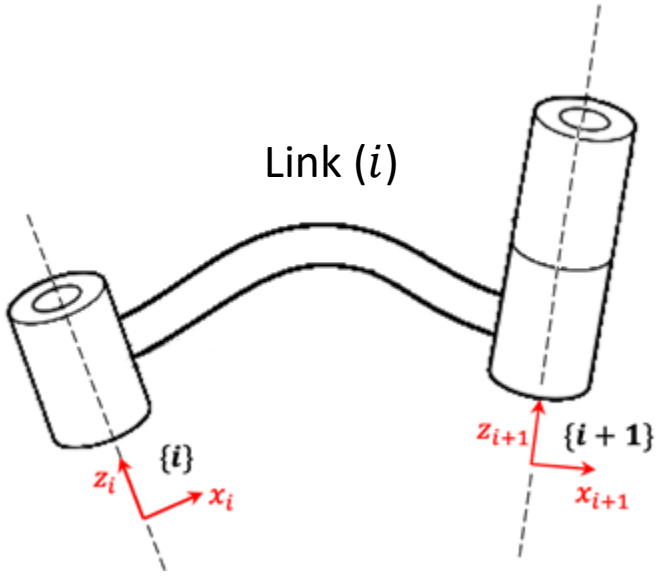
Link Velocity

If joint $(i+1)$ is a **revolute joint**:

$$\left[{}^{i+1}v_{i+1}\right] = \left[{}^{i+1}_{i}R\right]\left(\left[{}^{i}v_{i}\right] + \left[S\left({}^{i}\omega_{i}\right)\right]\left[{}^{i}P_{i+1}\right]\right)$$

$$\left[{}^{i+1}\omega_{i+1}\right] = \left[{}^{i+1}_{i}R\right]\left[{}^{i}\omega_{i}\right] + {}^{i+1}\begin{bmatrix}0\\0\\\dot{\theta}_{i+1}\end{bmatrix}$$



If joint $(i+1)$ is a **prismatic joint**:

$$\left[{}^{i+1}v_{i+1}\right] = \left[{}^{i+1}_{i}R\right]\left(\left[{}^{i}v_{i}\right] + \left[S\left({}^{i}\omega_{i}\right)\right]\left[{}^{i}P_{i+1}\right]\right) + {}^{i+1}\begin{bmatrix}0\\0\\\dot{d}_{i+1}\end{bmatrix}$$

$$\left[{}^{i+1}\omega_{i+1}\right] = \left[{}^{i+1}_{i}R\right]\left[{}^{i}\omega_{i}\right]$$

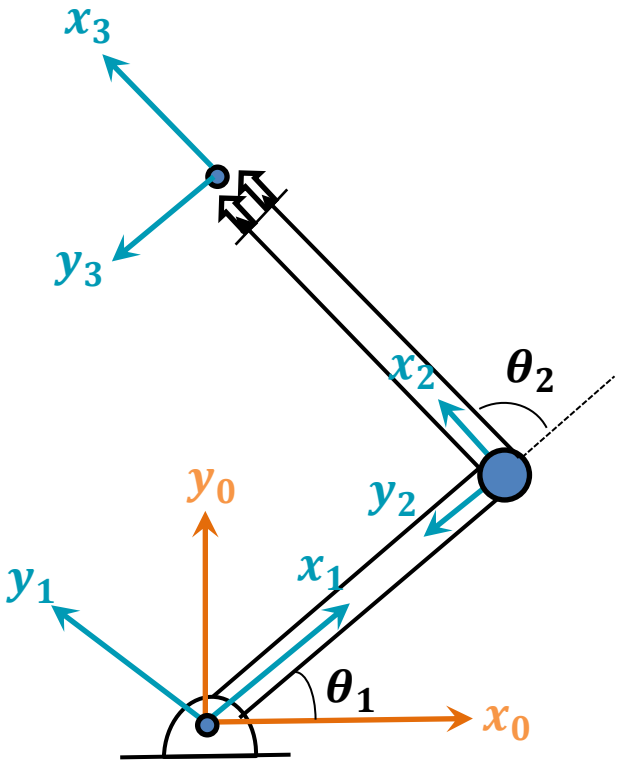
Link Velocity

Example: For the planar 2 DOF RR robotic arm, calculate the velocity of each link and that of the end-effector as a function of the joint rates?

$$[{}^{0}_{1}\mathrm{T}] = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[{}^{1}_{2}\mathrm{T}] = \begin{bmatrix} c_2 & -s_2 & 0 & l_1 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[{}^{2}_{3}\mathrm{T}] = \begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Link Velocity

$$[{}^0_1\mathrm{T}] = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow [{}^0_1R] = \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[{}^1_2\mathrm{T}] = \begin{bmatrix} c_2 & -s_2 & 0 & l_1 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow [{}^1_2R] = \begin{bmatrix} c_2 & -s_2 & 0 \\ s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[{}^2_3\mathrm{T}] = \begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow [{}^2_3R] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Link Velocity

$$[{}^0_1\mathrm{T}] = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow [{}^0P_1] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$[{}^1_2\mathrm{T}] = \begin{bmatrix} c_2 & -s_2 & 0 & l_1 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow [{}^1P_2] = \begin{bmatrix} l_1 \\ 0 \\ 0 \end{bmatrix}$$

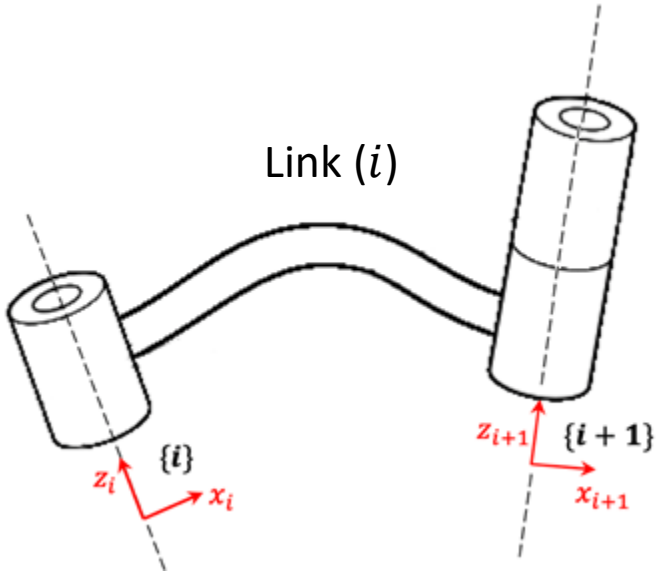
$$[{}^2_3\mathrm{T}] = \begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow [{}^2P_3] = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix}$$

Link Velocity

If joint $(i+1)$ is a **revolute joint**:

$$\left[{}^{i+1}v_{i+1}\right] = \left[{}^{i+1}_{i}R\right]\left(\left[{}^{i}v_{i}\right] + \left[S\left({}^{i}\omega_{i}\right)\right]\left[{}^{i}P_{i+1}\right]\right)$$

$$\left[{}^{i+1}\omega_{i+1}\right] = \left[{}^{i+1}_{i}R\right]\left[{}^{i}\omega_{i}\right] + {}^{i+1}\begin{bmatrix}0\\0\\\dot{\theta}_{i+1}\end{bmatrix}$$



If joint $(i+1)$ is a **prismatic joint**:

$$\left[{}^{i+1}v_{i+1}\right] = \left[{}^{i+1}_{i}R\right]\left(\left[{}^{i}v_{i}\right] + \left[S\left({}^{i}\omega_{i}\right)\right]\left[{}^{i}P_{i+1}\right]\right) + {}^{i+1}\begin{bmatrix}0\\0\\\dot{d}_{i+1}\end{bmatrix}$$

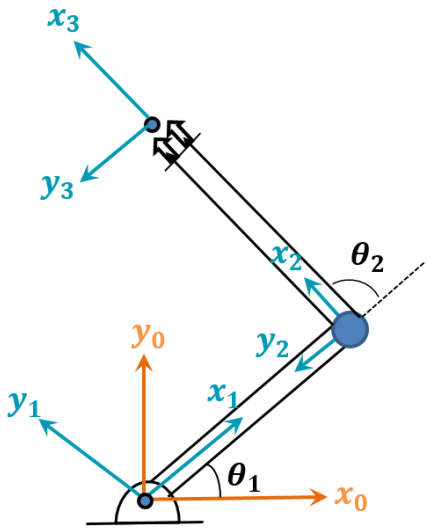
$$\left[{}^{i+1}\omega_{i+1}\right] = \left[{}^{i+1}_{i}R\right]\left[{}^{i}\omega_{i}\right]$$

Link Velocity

If joint $(i+1)$ is a **revolute joint**:

$$\left[{}^{i+1}v_{i+1}\right] = \left[{}^{i+1}_{i}R\right]\left(\left[{}^{i}v_{i}\right] + \left[S\left({}^{i}\omega_{i}\right)\right]\left[{}^{i}P_{i+1}\right]\right)$$

$$\left[{}^{i+1}\omega_{i+1}\right] = \left[{}^{i+1}_{i}R\right]\left[{}^{i}\omega_{i}\right] + {}^{i+1}\begin{bmatrix}0\\0\\\dot{\theta}_{i+1}\end{bmatrix}$$



$$\left[{}^{0}_{1}R\right] = \begin{bmatrix}c_1 & -s_1 & 0\\ s_1 & c_1 & 0\\ 0 & 0 & 1\end{bmatrix};\ \left[{}^{1}_{2}R\right] = \begin{bmatrix}c_2 & -s_2 & 0\\ s_2 & c_2 & 0\\ 0 & 0 & 1\end{bmatrix};\ \left[{}^{2}_{3}R\right] = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$\left[{}^{0}P_{1}\right] = \begin{bmatrix}0\\0\\0\end{bmatrix};\ \left[{}^{1}P_{2}\right] = \begin{bmatrix}l_1\\0\\0\end{bmatrix};\ \left[{}^{2}P_{3}\right] = \begin{bmatrix}l_2\\0\\0\end{bmatrix}$$

Link Velocity

For $i = 0$;

$$[{}^{1}\omega_1] = [{}^{1}_{0}R][{}^{0}\omega_0] + \begin{bmatrix}0\\0\\\dot{\theta}_1\end{bmatrix}$$

$$[{}^{1}\omega_1] = \begin{bmatrix}c_1 & -s_1 & 0\\ s_1 & c_1 & 0\\ 0 & 0 & 1\end{bmatrix}^T \begin{bmatrix}0\\0\\0\end{bmatrix} + \begin{bmatrix}0\\0\\\dot{\theta}_1\end{bmatrix} = \begin{bmatrix}0\\0\\\dot{\theta}_1\end{bmatrix}$$

$$[{}^{1}v_1] = [{}^{1}_{0}R]([{}^{0}v_0] + [S({}^{0}\omega_0)][{}^{0}P_1])$$

$$[{}^{1}v_1] = \begin{bmatrix}c_1 & -s_1 & 0\\ s_1 & c_1 & 0\\ 0 & 0 & 1\end{bmatrix}^T \left(\begin{bmatrix}0\\0\\0\end{bmatrix} + \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}0\\0\\0\end{bmatrix}\right)$$

$$[{}^{1}v_1] = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$[{}^{i+1}\omega_{i+1}] = [{}^{i+1}_{i}R][{}^{i}\omega_i] + {}^{i+1}\begin{bmatrix}0\\0\\\dot{\theta}_{i+1}\end{bmatrix}$$

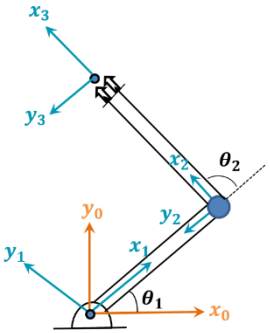
$$[{}^{i+1}v_{i+1}] = [{}^{i+1}_{i}R]([{}^{i}v_i] + [S({}^{i}\omega_i)][{}^{i}P_{i+1}])$$

$$[S({}^{i}\omega_i)] = \begin{bmatrix}0 & -\omega_z & \omega_y\\ \omega_z & 0 & -\omega_x\\ -\omega_y & \omega_x & 0\end{bmatrix}$$

$$[{}^{0}_{1}R] = \begin{bmatrix}c_1 & -s_1 & 0\\ s_1 & c_1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$[{}^{1}_{2}R] = \begin{bmatrix}c_2 & -s_2 & 0\\ s_2 & c_2 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$[{}^{2}_{3}R] = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$



$$[{}^{0}P_1] = \begin{bmatrix}0\\0\\0\end{bmatrix} \;;\; [{}^{1}P_2] = \begin{bmatrix}l_1\\0\\0\end{bmatrix} \;;\; [{}^{2}P_3] = \begin{bmatrix}l_2\\0\\0\end{bmatrix}$$

Link Velocity

$$\left[{}^{1}\omega_1\right] = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} \quad \text{and} \quad \left[{}^{1}v_1\right] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

For $i = 1$;

$$\left[{}^{2}\omega_2\right] = \left[{}^{2}_{1}R\right]\left[{}^{1}\omega_1\right] + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_2 \end{bmatrix}$$

$$\left[{}^{2}\omega_2\right] = \begin{bmatrix} c_2 & -s_2 & 0 \\ s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{T} \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_2 \end{bmatrix}$$

$$\left[{}^{2}\omega_2\right] = \begin{bmatrix} c_2 & s_2 & 0 \\ -s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_2 \end{bmatrix}$$

$$\left[{}^{2}\omega_2\right] = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 + \dot{\theta}_2 \end{bmatrix}$$

$$\left[{}^{i+1}\omega_{i+1}\right] = \left[{}^{i+1}_{i}R\right]\left[{}^{i}\omega_i\right] + {}^{i+1}\begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_{i+1} \end{bmatrix}$$

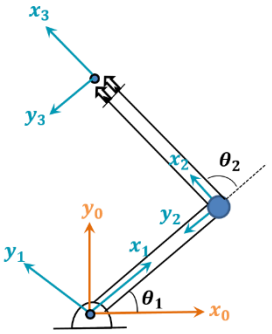
$$\left[{}^{i+1}v_{i+1}\right] = \left[{}^{i+1}_{i}R\right]\left(\left[{}^{i}v_i\right] + \left[S\left({}^{i}\omega_i\right)\right]\left[{}^{i}P_{i+1}\right]\right)$$

$$\left[S\left({}^{i}\omega_i\right)\right] = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}$$

$$\left[{}^{0}_{1}R\right] = \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\left[{}^{1}_{2}R\right] = \begin{bmatrix} c_2 & -s_2 & 0 \\ s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\left[{}^{2}_{3}R\right] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



$$\left[{}^{0}P_1\right] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \; ; \; \left[{}^{1}P_2\right] = \begin{bmatrix} l_1 \\ 0 \\ 0 \end{bmatrix} \; ; \; \left[{}^{2}P_3\right] = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix}$$

Link Velocity

$$[{}^1\omega_1] = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix};\ [{}^1v_1] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix};\ [{}^2\omega_2] = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 + \dot{\theta}_2 \end{bmatrix}$$

Continue for $i = 1$;

$$[{}^2v_2] = [{}^2_1R]([{}^1v_1] + [S({}^1\omega_1)][{}^1P_2])$$

$$[{}^2v_2] = \begin{bmatrix} c_2 & -s_2 & 0 \\ s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}^T \left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & -\dot{\theta}_1 & 0 \\ \dot{\theta}_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} l_1 \\ 0 \\ 0 \end{bmatrix} \right)$$

$$[{}^2v_2] = \begin{bmatrix} c_2 & s_2 & 0 \\ -s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ \dot{\theta}_1 l_1 \\ 0 \end{bmatrix}$$

$$[{}^2v_2] = \begin{bmatrix} \dot{\theta}_1 l_1 s_2 \\ \dot{\theta}_1 l_1 c_2 \\ 0 \end{bmatrix}$$

$$[{}^{i+1}\omega_{i+1}] = [{}^{i+1}_iR][{}^i\omega_i] + {}^{i+1}\begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_{i+1} \end{bmatrix}$$

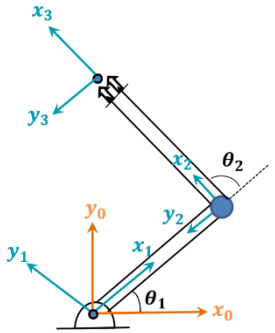
$$[{}^{i+1}v_{i+1}] = [{}^{i+1}_iR]([{}^iv_i] + [S({}^i\omega_i)][{}^iP_{i+1}])$$

$$[S({}^i\omega_i)] = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}$$

$$[{}^0_1R] = \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[{}^1_2R] = \begin{bmatrix} c_2 & -s_2 & 0 \\ s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[{}^2_3R] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



$$[{}^0P_1] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix};\ [{}^1P_2] = \begin{bmatrix} l_1 \\ 0 \\ 0 \end{bmatrix};\ [{}^2P_3] = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix}$$

Link Velocity

$$[{}^1\omega_1] = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix};\ [{}^1v_1] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$[{}^2\omega_2] = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 + \dot{\theta}_2 \end{bmatrix};\ [{}^2v_2] = \begin{bmatrix} \dot{\theta}_1 l_1 s_2 \\ \dot{\theta}_1 l_1 c_2 \\ 0 \end{bmatrix}$$

For $i = 2$;

$$[{}^3\omega_3] = [{}^3_2R][{}^2\omega_2] + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$[{}^3\omega_3] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^T \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 + \dot{\theta}_2 \end{bmatrix}$$

$$[{}^3\omega_3] = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 + \dot{\theta}_2 \end{bmatrix}$$

$$[{}^{i+1}\omega_{i+1}] = [{}^{i+1}_iR][{}^i\omega_i] + {}^{i+1}\begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_{i+1} \end{bmatrix}$$

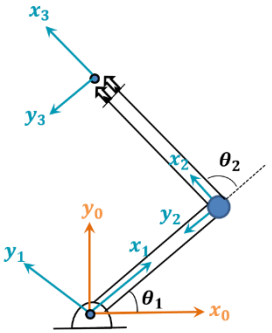
$$[{}^{i+1}v_{i+1}] = [{}^{i+1}_iR]([{}^iv_i] + [S({}^i\omega_i)][{}^iP_{i+1}])$$

$$[S({}^i\omega_i)] = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}$$

$$[{}^0_1R] = \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[{}^1_2R] = \begin{bmatrix} c_2 & -s_2 & 0 \\ s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[{}^2_3R] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



$$[{}^0P_1] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix};\ [{}^1P_2] = \begin{bmatrix} l_1 \\ 0 \\ 0 \end{bmatrix};\ [{}^2P_3] = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix}$$

Link Velocity

$$[{}^1\omega_1] = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix};\ [{}^1v_1] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix};\ [{}^2\omega_2] = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 + \dot{\theta}_2 \end{bmatrix}$$

$$[{}^2v_2] = \begin{bmatrix} \dot{\theta}_1 l_1 s_2 \\ \dot{\theta}_1 l_1 c_2 \\ 0 \end{bmatrix};\ [{}^3\omega_3] = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 + \dot{\theta}_2 \end{bmatrix}$$

Continue for $i = 2$;

$$[{}^3v_3] = [{}^3_2R]([{}^2v_2] + [S({}^2\omega_2)][{}^2P_3])$$

$$[{}^3v_3] = [I]\left(\begin{bmatrix} \dot{\theta}_1 l_1 s_2 \\ \dot{\theta}_1 l_1 c_2 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & -(\dot{\theta}_1+\dot{\theta}_2) & 0 \\ (\dot{\theta}_1+\dot{\theta}_2) & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix}\right)$$

$$[{}^3v_3] = \begin{bmatrix} \dot{\theta}_1 l_1 s_2 \\ \dot{\theta}_1 l_1 c_2 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ (\dot{\theta}_1 + \dot{\theta}_2) l_2 \\ 0 \end{bmatrix} = \begin{bmatrix} \dot{\theta}_1 l_1 s_2 \\ \dot{\theta}_1 l_1 c_2 + (\dot{\theta}_1+\dot{\theta}_2) l_2 \\ 0 \end{bmatrix}$$

$$[{}^{i+1}\omega_{i+1}] = [{}^{i+1}_iR][{}^i\omega_i] + {}^{i+1}\begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_{i+1} \end{bmatrix}$$

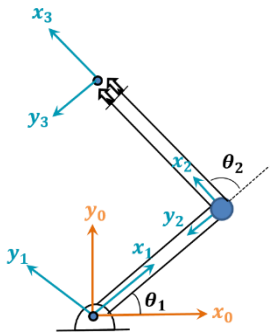
$$[{}^{i+1}v_{i+1}] = [{}^{i+1}_iR]([{}^iv_i] + [S({}^i\omega_i)][{}^iP_{i+1}])$$

$$[S({}^i\omega_i)] = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}$$

$$[{}^0_1R] = \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[{}^1_2R] = \begin{bmatrix} c_2 & -s_2 & 0 \\ s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[{}^2_3R] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



$$[{}^0P_1] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix};\ [{}^1P_2] = \begin{bmatrix} l_1 \\ 0 \\ 0 \end{bmatrix};\ [{}^2P_3] = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix}$$

Link Velocity

$$[{}^1\omega_1] = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix}$$

$$[{}^1v_1] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$[{}^2\omega_2] = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 + \dot{\theta}_2 \end{bmatrix}$$

$$[{}^2v_2] = \begin{bmatrix} \dot{\theta}_1 l_1 s_2 \\ \dot{\theta}_1 l_1 c_2 \\ 0 \end{bmatrix}$$

$$[{}^3\omega_3] = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 + \dot{\theta}_2 \end{bmatrix}$$

$$[{}^3v_3] = \begin{bmatrix} \dot{\theta}_1 l_1 s_2 \\ \dot{\theta}_1 l_1 c_2 + (\dot{\theta}_1 + \dot{\theta}_2) l_2 \\ 0 \end{bmatrix}$$

$$[{}^{i+1}\omega_{i+1}] = [{}^{i+1}_{i}R][{}^{i}\omega_i] + {}^{i+1}\begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_{i+1} \end{bmatrix}$$

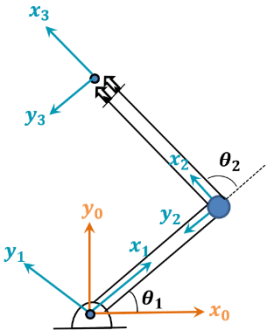
$$[{}^{i+1}v_{i+1}] = [{}^{i+1}_{i}R]([{}^{i}v_i] + [S({}^{i}\omega_i)][{}^{i}P_{i+1}])$$

$$[S({}^{i}\omega_i)] = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}$$

$$[{}^0_1R] = \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[{}^1_2R] = \begin{bmatrix} c_2 & -s_2 & 0 \\ s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[{}^2_3R] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



$$[{}^0P_1] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \; ; \; [{}^1P_2] = \begin{bmatrix} l_1 \\ 0 \\ 0 \end{bmatrix} \; ; \; [{}^2P_3] = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix}$$

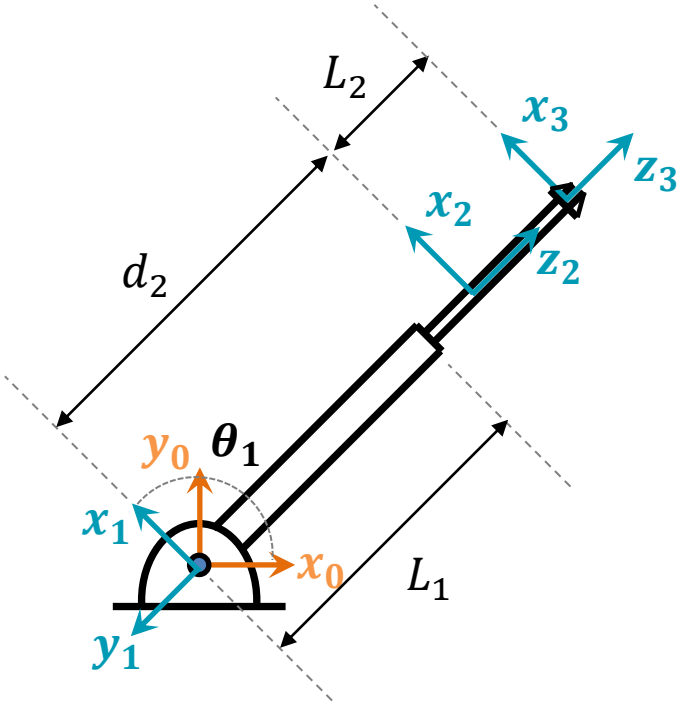
Link Velocity

Example: For the planar 2 DOF RP robotic arm, calculate the velocity of each link and that of the end-effector as a function of the joint rates?

$$[{}^{0}_{1}\mathrm{T}] = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[{}^{1}_{2}\mathrm{T}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -d_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[{}^{2}_{3}\mathrm{T}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Link Velocity

$$[{}^0_1\mathrm{T}] = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow [{}^0_1R] = \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[{}^1_2\mathrm{T}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -d_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow [{}^1_2R] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$[{}^2_3\mathrm{T}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow [{}^2_3R] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Link Velocity

$$[{}^0_1\mathrm{T}] = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow [{}^0P_1] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$[{}^1_2\mathrm{T}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -d_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow [{}^1P_2] = \begin{bmatrix} 0 \\ -d_2 \\ 0 \end{bmatrix}$$

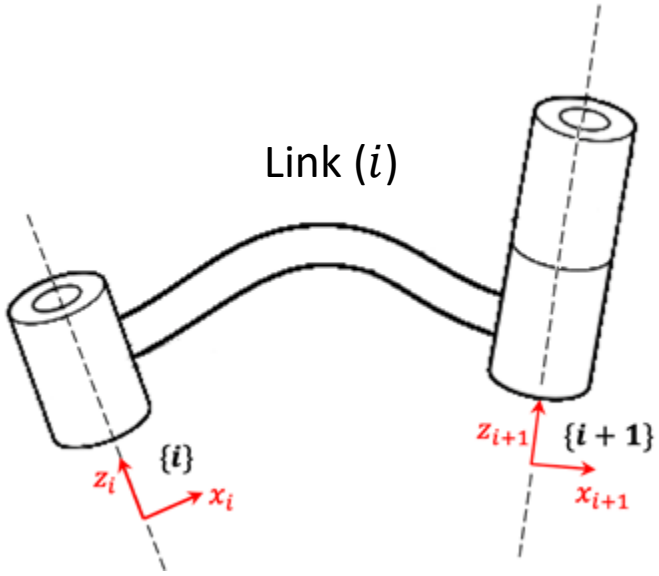
$$[{}^2_3\mathrm{T}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow [{}^2P_3] = \begin{bmatrix} 0 \\ 0 \\ L_2 \end{bmatrix}$$

Link Velocity

If joint $(i+1)$ is a **revolute joint**:

$$\left[{}^{i+1}v_{i+1}\right] = \left[{}^{i+1}_{i}R\right]\left(\left[{}^{i}v_{i}\right] + \left[S\left({}^{i}\omega_{i}\right)\right]\left[{}^{i}P_{i+1}\right]\right)$$

$$\left[{}^{i+1}\omega_{i+1}\right] = \left[{}^{i+1}_{i}R\right]\left[{}^{i}\omega_{i}\right] + {}^{i+1}\begin{bmatrix}0\\0\\\dot{\theta}_{i+1}\end{bmatrix}$$



If joint $(i+1)$ is a **prismatic joint**:

$$\left[{}^{i+1}v_{i+1}\right] = \left[{}^{i+1}_{i}R\right]\left(\left[{}^{i}v_{i}\right] + \left[S\left({}^{i}\omega_{i}\right)\right]\left[{}^{i}P_{i+1}\right]\right) + {}^{i+1}\begin{bmatrix}0\\0\\\dot{d}_{i+1}\end{bmatrix}$$

$$\left[{}^{i+1}\omega_{i+1}\right] = \left[{}^{i+1}_{i}R\right]\left[{}^{i}\omega_{i}\right]$$

Link Velocity

For $i = 0$:

$$[{}^1\omega_1] = [{}^1_0R][{}^0\omega_0] + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix}$$

$$[{}^1\omega_1] = \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^T \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix}$$

$$[{}^1\omega_1] = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix}$$

$$[{}^1v_1] = [{}^1_0R]([{}^0v_0] + [S({}^0\omega_0)][{}^0P_1])$$

$$[{}^1v_1] = \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^T \left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right)$$

$$[{}^1v_1] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

If joint $(i+1)$ is a **revolute joint**:

$$[{}^{i+1}\omega_{i+1}] = [{}^{i+1}_iR][{}^i\omega_i] + {}^{i+1}\begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_{i+1} \end{bmatrix}$$

$$[{}^{i+1}v_{i+1}] = [{}^{i+1}_iR]([{}^iv_i] + [S({}^i\omega_i)][{}^iP_{i+1}])$$

If joint $(i+1)$ is a **prismatic joint**:

$$[{}^{i+1}v_{i+1}] = [{}^{i+1}_iR]([{}^iv_i] + [S({}^i\omega_i)][{}^iP_{i+1}]) + {}^{i+1}\begin{bmatrix} 0 \\ 0 \\ \dot{d}_{i+1} \end{bmatrix}$$

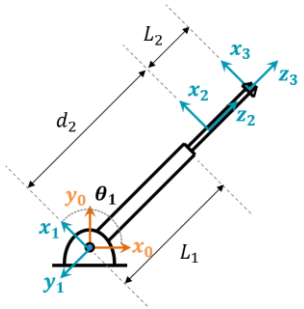
$$[{}^{i+1}\omega_{i+1}] = [{}^{i+1}_iR][{}^i\omega_i]$$

$$[S({}^i\omega_i)] = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}$$

$$[{}^0_1R] = \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[{}^1_2R] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$[{}^2_3R] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



$$[{}^0P_1] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \; ; \; [{}^1P_2] = \begin{bmatrix} 0 \\ -d_2 \\ 0 \end{bmatrix} \; ; \; [{}^2P_3] = \begin{bmatrix} 0 \\ 0 \\ L_2 \end{bmatrix}$$

Link Velocity

$$[{}^1\omega_1] = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} \quad \text{and} \quad [{}^1v_1] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

For $i = 1$:

$$[{}^2\omega_2] = [{}^2_1R][{}^1\omega_1]$$

$$[{}^2\omega_2] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}^T \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix}$$

$$[{}^2\omega_2] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix}$$

$$[{}^2\omega_2] = \begin{bmatrix} 0 \\ \dot{\theta}_1 \\ 0 \end{bmatrix}$$

If joint $(i+1)$ is a **revolute joint**:

$$[{}^{i+1}\omega_{i+1}] = [{}^{i+1}_iR][{}^i\omega_i] + {}^{i+1}\begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_{i+1} \end{bmatrix}$$

$$[{}^{i+1}v_{i+1}] = [{}^{i+1}_iR]([{}^iv_i] + [S({}^i\omega_i)][{}^iP_{i+1}])$$

If joint $(i+1)$ is a **prismatic joint**:

$$[{}^{i+1}v_{i+1}] = [{}^{i+1}_iR]([{}^iv_i] + [S({}^i\omega_i)][{}^iP_{i+1}]) + {}^{i+1}\begin{bmatrix} 0 \\ 0 \\ \dot{d}_{i+1} \end{bmatrix}$$

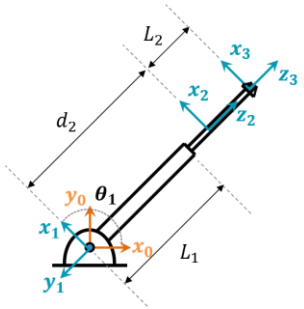
$$[{}^{i+1}\omega_{i+1}] = [{}^{i+1}_iR][{}^i\omega_i]$$

$$[S({}^i\omega_i)] = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}$$

$$[{}^0_1R] = \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[{}^1_2R] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$[{}^2_3R] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



$$[{}^0P_1] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \; ; \; [{}^1P_2] = \begin{bmatrix} 0 \\ -d_2 \\ 0 \end{bmatrix} \; ; \; [{}^2P_3] = \begin{bmatrix} 0 \\ 0 \\ L_2 \end{bmatrix}$$

Link Velocity

$$[{}^1\omega_1] = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} \; ; \; [{}^1v_1] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \; ; \; [{}^2\omega_2] = \begin{bmatrix} 0 \\ \dot{\theta}_1 \\ 0 \end{bmatrix}$$

Continue for $i = 1$:

$$[{}^2v_2] = [{}^2_1R]([{}^1v_1] + [S({}^1\omega_1)][{}^1P_2]) + {}^2\begin{bmatrix} 0 \\ 0 \\ \dot{d}_2 \end{bmatrix}$$

$$[{}^2v_2] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}^T \left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & -\dot{\theta}_1 & 0 \\ \dot{\theta}_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ -d_2 \\ 0 \end{bmatrix} \right) + \begin{bmatrix} 0 \\ 0 \\ \dot{d}_2 \end{bmatrix}$$

$$[{}^2v_2] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 d_2 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{d}_2 \end{bmatrix}$$

$$[{}^2v_2] = \begin{bmatrix} \dot{\theta}_1 d_2 \\ 0 \\ \dot{d}_2 \end{bmatrix}$$

If joint $(i+1)$ is a **revolute joint**:

$$[{}^{i+1}\omega_{i+1}] = [{}^{i+1}_iR][{}^i\omega_i] + {}^{i+1}\begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_{i+1} \end{bmatrix}$$

$$[{}^{i+1}v_{i+1}] = [{}^{i+1}_iR]([{}^iv_i] + [S({}^i\omega_i)][{}^iP_{i+1}])$$

If joint $(i+1)$ is a **prismatic joint**:

$$[{}^{i+1}v_{i+1}] = [{}^{i+1}_iR]([{}^iv_i] + [S({}^i\omega_i)][{}^iP_{i+1}]) + {}^{i+1}\begin{bmatrix} 0 \\ 0 \\ \dot{d}_{i+1} \end{bmatrix}$$

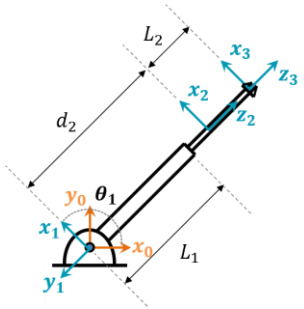
$$[{}^{i+1}\omega_{i+1}] = [{}^{i+1}_iR][{}^i\omega_i]$$

$$[S({}^i\omega_i)] = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}$$

$$[{}^0_1R] = \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[{}^1_2R] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$[{}^2_3R] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



$$[{}^0P_1] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \; ; \; [{}^1P_2] = \begin{bmatrix} 0 \\ -d_2 \\ 0 \end{bmatrix} \; ; \; [{}^2P_3] = \begin{bmatrix} 0 \\ 0 \\ L_2 \end{bmatrix}$$

Link Velocity

$$[\,{}^1\omega_1] = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} \; ; \; [\,{}^1v_1] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$[\,{}^2\omega_2] = \begin{bmatrix} 0 \\ \dot{\theta}_1 \\ 0 \end{bmatrix} \; ; \; [\,{}^2v_2] = \begin{bmatrix} \dot{\theta}_1 d_2 \\ 0 \\ \dot{d}_2 \end{bmatrix}$$

For $i = 2$:

$$[\,{}^3\omega_3] = [{}^3_2R][\,{}^2\omega_2]$$

$$[\,{}^3\omega_3] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^T \begin{bmatrix} 0 \\ \dot{\theta}_1 \\ 0 \end{bmatrix}$$

$$[\,{}^3\omega_3] = \begin{bmatrix} 0 \\ \dot{\theta}_1 \\ 0 \end{bmatrix}$$

If joint $(i+1)$ is a **revolute joint**:

$$[\,{}^{i+1}\omega_{i+1}] = [{}^{i+1}_iR][\,{}^i\omega_i] + {}^{i+1}\begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_{i+1} \end{bmatrix}$$

$$[\,{}^{i+1}v_{i+1}] = [{}^{i+1}_iR]([\,{}^iv_i] + [S(\,{}^i\omega_i)][\,{}^iP_{i+1}])$$

If joint $(i+1)$ is a **prismatic joint**:

$$[\,{}^{i+1}v_{i+1}] = [{}^{i+1}_iR]([{}^iv_i] + [S({}^i\omega_i)][{}^iP_{i+1}]) + {}^{i+1}\begin{bmatrix} 0 \\ 0 \\ \dot{d}_{i+1} \end{bmatrix}$$

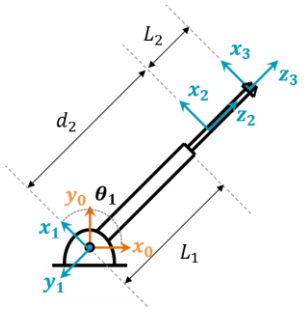
$$[\,{}^{i+1}\omega_{i+1}] = [{}^{i+1}_iR][{}^i\omega_i]$$

$$[S(\,{}^i\omega_i)] = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}$$

$$[{}^0_1R] = \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[{}^1_2R] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$[{}^2_3R] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



$$[{}^0P_1] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \; ; \; [{}^1P_2] = \begin{bmatrix} 0 \\ -d_2 \\ 0 \end{bmatrix} \; ; \; [{}^2P_3] = \begin{bmatrix} 0 \\ 0 \\ L_2 \end{bmatrix}$$

Link Velocity

$$[\,^1\omega_1] = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} \; ; \; [\,^1v_1] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \; ; \; [\,^2\omega_2] = \begin{bmatrix} 0 \\ \dot{\theta}_1 \\ 0 \end{bmatrix}$$

$$[\,^2v_2] = \begin{bmatrix} \dot{\theta}_1 d_2 \\ 0 \\ \dot{d}_2 \end{bmatrix} \; ; \; [\,^3\omega_3] = \begin{bmatrix} 0 \\ \dot{\theta}_1 \\ 0 \end{bmatrix}$$

Continue for $i = 2$:

$$[\,^3v_3] = [\,^3_2R]([\,^2v_2] + [S(\,^2\omega_2)][\,^2P_3])$$

$$[\,^3v_3] = [I]\left(\begin{bmatrix} \dot{\theta}_1 d_2 \\ 0 \\ \dot{d}_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 & \dot{\theta}_1 \\ 0 & 0 & 0 \\ -\dot{\theta}_1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ L_2 \end{bmatrix}\right)$$

$$[\,^3v_3] = \begin{bmatrix} \dot{\theta}_1 d_2 \\ 0 \\ \dot{d}_2 \end{bmatrix} + \begin{bmatrix} \dot{\theta}_1 L_2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \dot{\theta}_1 (d_2 + L_2) \\ 0 \\ \dot{d}_2 \end{bmatrix}$$

If joint $(i+1)$ is a **revolute joint**:

$$[\,^{i+1}\omega_{i+1}] = [\,^{i+1}_iR][\,^i\omega_i] + {}^{i+1}\begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_{i+1} \end{bmatrix}$$

$$[\,^{i+1}v_{i+1}] = [\,^{i+1}_iR]([\,^iv_i] + [S(\,^i\omega_i)][\,^iP_{i+1}])$$

If joint $(i+1)$ is a **prismatic joint**:

$$[\,^{i+1}v_{i+1}] = [\,^{i+1}_iR]([\,^iv_i] + [S(\,^i\omega_i)][\,^iP_{i+1}]) + {}^{i+1}\begin{bmatrix} 0 \\ 0 \\ \dot{d}_{i+1} \end{bmatrix}$$

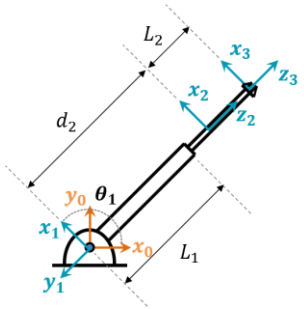
$$[\,^{i+1}\omega_{i+1}] = [\,^{i+1}_iR][\,^i\omega_i]$$

$$[S(\,^i\omega_i)] = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}$$

$$[\,^0_1R] = \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[\,^1_2R] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$[\,^2_3R] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



$$[\,^0P_1] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \; ; \; [\,^1P_2] = \begin{bmatrix} 0 \\ -d_2 \\ 0 \end{bmatrix} \; ; \; [\,^2P_3] = \begin{bmatrix} 0 \\ 0 \\ L_2 \end{bmatrix}$$

Link Velocity

$$[{}^1\omega_1] = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix}$$

$$[{}^1v_1] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$[{}^2\omega_2] = \begin{bmatrix} 0 \\ \dot{\theta}_1 \\ 0 \end{bmatrix}$$

$$[{}^2v_2] = \begin{bmatrix} \dot{\theta}_1 d_2 \\ 0 \\ \dot{d}_2 \end{bmatrix}$$

$$[{}^3\omega_3] = \begin{bmatrix} 0 \\ \dot{\theta}_1 \\ 0 \end{bmatrix}$$

$$[{}^3v_3] = \begin{bmatrix} \dot{\theta}_1 (d_2 + L_2) \\ 0 \\ \dot{d}_2 \end{bmatrix}$$

If joint $(i+1)$ is a **revolute joint**:

$$[{}^{i+1}\omega_{i+1}] = [{}^{i+1}_{i}R][{}^{i}\omega_i] + {}^{i+1}\begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_{i+1} \end{bmatrix}$$

$$[{}^{i+1}v_{i+1}] = [{}^{i+1}_{i}R]([{}^{i}v_i] + [S({}^{i}\omega_i)][{}^{i}P_{i+1}])$$

If joint $(i+1)$ is a **prismatic joint**:

$$[{}^{i+1}v_{i+1}] = [{}^{i+1}_{i}R]([{}^{i}v_i] + [S({}^{i}\omega_i)][{}^{i}P_{i+1}]) + {}^{i+1}\begin{bmatrix} 0 \\ 0 \\ \dot{d}_{i+1} \end{bmatrix}$$

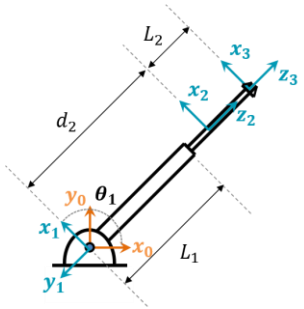
$$[{}^{i+1}\omega_{i+1}] = [{}^{i+1}_{i}R][{}^{i}\omega_i]$$

$$[S({}^{i}\omega_i)] = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}$$

$$[{}^0_1R] = \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[{}^1_2R] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$[{}^2_3R] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



$$[{}^0P_1] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \; ; \; [{}^1P_2] = \begin{bmatrix} 0 \\ -d_2 \\ 0 \end{bmatrix} \; ; \; [{}^2P_3] = \begin{bmatrix} 0 \\ 0 \\ L_2 \end{bmatrix}$$

Link Velocity

For more details on this subject, please see:

- Introduction to Robotics: Mechanics and Control, by John J. Craig, 3rd Edition, Addison-Wesley Publishing Company, 2003.